

Assessment of the Stability of a Four Legged Robot Manipulator

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Abstract: This paper deals with static stability of a four legged robot manipulator when it moves with trot gait. In this paper, the stability analysis based on dynamics of walking usually proposed for the analysis of walking system studied and identified its limits and ambiguity. After identifying the key issues of stability, it has been identified that moment around the supporting diagonal line of quadruped in trotting gait largely influences walking stability. Therefore Moment around the supporting diagonal line of quadruped in trotting gait is modeled and its effects on body attitude (roll and pitch) are analyzed. The degree of influence varies with different initial stances of quadruped and we get the optimal initial stance of quadruped in trotting gait with maximal walking stability. Simulation results are presented.

Key words: Platform, body, feet, legs, manipulator, trot, stride length, stability.

I. INTRODUCTION

For applications such as planetary or volcanic exploration or various missions in hazardous areas or construction sites, Walking mobile robots are very essential. The fact that a manipulator is mounted on a mobile platform greatly increases its workspace, allowing for a variety of applications such as robots for bomb squadron, search and rescue and undersea work, as well as unmanned vehicles and robotic cameras. As a kind of legged vehicles, quadruped robots with manipulators can be applied in such environment. The locomotion performances in terms of power consumption, autonomy and stability are of first importance. Vehicle motion on uneven surfaces involves complex ground interactions that are related to balance and stability of the vehicle. Therefore, enhancing the locomotion performances in such environment requires the design of innovative locomotion systems and the research of original control schemes.

Walking and running machines are classified into two categories based on the stability of their motion: passively stable and dynamically stable. For a passively stable system, the vertical projection of the center of mass always remains within the closed region formed by the contact points of the feet on the ground. This region is called the support polygon. Fig.1 They typically have four or six legs but may be bipeds with large feet surface area. In contrast, dynamically stable systems utilize dynamic forces and feedback to maintain balance. Dynamically stable machines have been built with one, two, and four legs. Because dynamically stable systems are more difficult to design, control and analyze, the early legged robots were mostly statically stable machines. The gait pattern of a walking vehicle plays important role in walking stability. The most critical gait of a four legged walking vehicle is trot.

Quadruped machine walking in high speed requires dynamic walking with lower duty factor. Trotting gait is one of typical dynamic walking patterns. In trotting gait, two pairs of diagonal legs make standing phase respectively. Thus, there exists a moment when the body is falling down around the supporting axis, which makes the body attitude vary. Pierre-Brice WIEBER et al [i] proposed to determine falling down moment by considering the dynamics of vehicle which has been discussed in section 2. After considering the problems of dynamics model, we proceeded to a new model.

The moment falling down around the supporting axis is modeled considering both sideways direction and lengthwise direction, and the angle of rotation around the supporting diagonal line is derived in the section 3 with the help of concepts developed by Dongqing He& Peisun Ma [ii].

Next, the effects of the moment on body attitude and walking stability are analyzed. The analysis indicates that the degree of influence varies with different initial stances of quadruped [iii][iv]. Finally we get the optimal initial stance of quadruped in trotting gait with maximal walking stability and simulation results are presented in section 4.

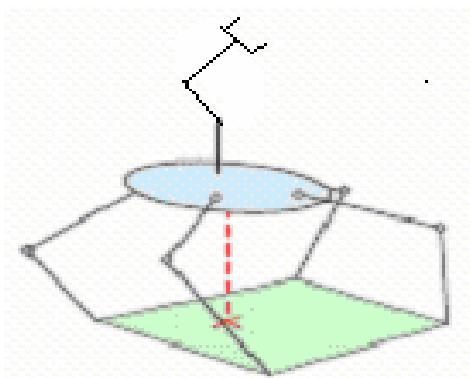


Fig (1): Walking with manipulator

II. DYNAMICS OF WALKING

As long as generic walking systems are considered (no thrusters, for example), exterior forces are solely gravity and contact forces, and gravity being unalterable, appropriate contact forces will be the only way for the system to effectively realize any specific movement [v].

When walking systems are systems of articulated rigid bodies, their complete dynamics can be written as a classical set of Euler-Lagrange equations

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} + G(q) = T(q)u + C(q)^T \quad (1)$$

Where q is the configuration vector which has to account for two different information: the shape of the system that can be described by the joint positions q^1 , and its position and orientation in the space, that can be described by the

position and orientation q^2 of a manipulator attached to some part of the system. The vector q manifesting a structure

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$T(q)u$ are the actuation forces and $c(q)^T\lambda$ contact forces. we can split the dynamics (2) to exhibit the same structure:

$$\begin{bmatrix} M_1(q) \\ M_2(q) \end{bmatrix} \ddot{q} + \begin{bmatrix} N_1(q, \dot{q}) \\ N_2(q, \dot{q}) \end{bmatrix} \dot{q} + \begin{bmatrix} C_1(q) \\ C_2(q) \end{bmatrix} = \dots \dots \begin{bmatrix} T_1(q) \\ 0 \end{bmatrix} u + \begin{bmatrix} C_1(q)^T \\ C_2(q)^T \end{bmatrix} \lambda \quad (2)$$

A walking system can realize a movement $q(t)$ if and only if equation (1) is satisfied with appropriate actuation and contact forces $u(t)$ and $\lambda(t)$. Now, whatever the possibilities of the actuation forces, the lower part (2) has to be satisfied with the only action of contact forces, and the physics of contact is such that these forces have limitations: in the general case (no gluing, for example), contacting solids can push one another but they can't pull one another (what is referred to as the unilaterality of contacts), and friction between them is limited [vi]. Considering this restriction of contact forces together with the lower part of the dynamics (2), a necessary condition for a walking system to realize a movement $q(t)$ is that there exist contact forces such the

$$M_2(q)\ddot{q} + N_2(q, \dot{q})\dot{q} + G_2(q) = C_2(q)^T\lambda \quad (3)$$

But this completely general criterion may be complex to deal with since it needs to answer the question: does there exist a λ such that (4) is satisfied.

Numerical methods stemming from optimization theory are able to answer to this kind of question, but at a computational expense that can be hindering. Hence we proposed a new model to assess the stability of such systems.

III. SIMPLIFIED MODEL

During walking of quadruped, the manipulator does not perform any task hence assumed a static. It is assumed that the overall mass of manipulator concentrated at e.g. (G) of walking platform. The effect of dynamics of two swing legs neglected in this model. At any instant, the body is supported by diagonal opposite legs [v][vi].

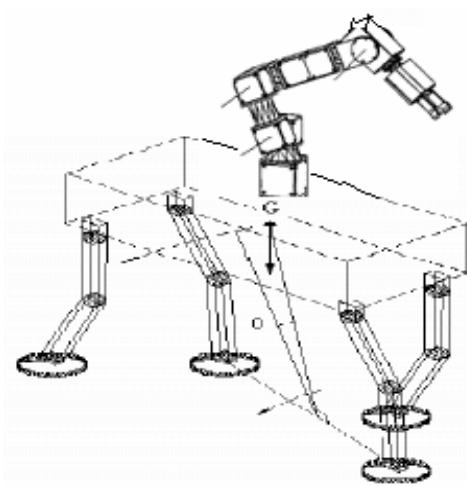


Fig (2): Quadruped robot with manipulator rotates around the supporting diagonal line

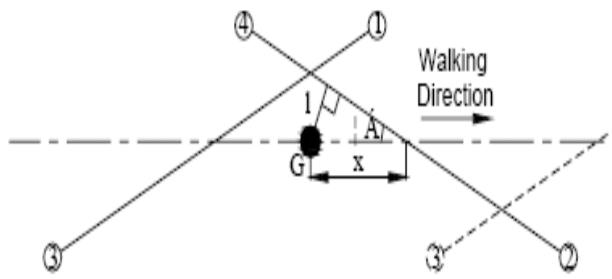


Fig (3): Projection of trot gait on terrain

As the projection of gravity G in the supporting plane is not on the supporting diagonal line, which causes a moment falling down around the supporting axis as illustrated in Fig.2(θ means the angel of rotation). Quadruped walking machine will rotate around the axis because of action of the moment. We want to learn how much it influences rotation and walking stability.

We assume the followings:

1. The two supporting legs on the supporting diagonal line always contact supporting ground in the 1st half of cycle period of walking and the robot only rotates around the axis without other rotations. In the 1st half of cycle period of walking, quadruped robot starts to walk and, there is no obvious collision with the ground. In other periods of walking, the swinging legs will touch ground with collision and the assumption is not true probably.
2. Four legs of the robot are massless and the mass of the robot and manipulator is concentrated on the body.
3. The action of joint torque on the body is neglected. It is significantly smaller compared to that of the rotating moment This assumption is very important
4. Quadruped machine walks with constant speed.
5. The terrain is rigid, regular and even.

In order to illuminate dynamics relations, according to previous assumptions, especially assumption 1, 2 and 3.

We have this equation by Euler's theorem in the 1st cycle period of walking [vii]:

$$mgl = \varepsilon I \quad (4)$$

where m is the mass of the quadruped walking machine, g is acceleration of gravity, l is the distance between the projection of center of gravity (COG) in the terrain and the supporting diagonal line, I is the moment of inertia of the robot along with manipulator and ε is angular acceleration of the rotation. It must be clarified again that above equation may be not true without assumption 1 to 3. As illustrated in Fig. 3, the distance in the walking direction from the projection of center of gravity in the terrain to the supporting diagonal line is x . To learn the relation between stride length S and x on walking stability, the variable x_1 ($0 \leq x_1 \leq 0.5$) is selected and x is expressed as $x_1 S$. The constant walking speed is v and l is a function of time of walking t because it decreases when center of gravity moves along the walking direction. According to Fig. 3, we have:

$$I(t) = (x_1 S - \int v dt) \sin \alpha \quad (5)$$

where S is stride length of the robot, α is angle between walking direction and the supporting diagonal

Substituting (5) into (4), we obtain: $\varepsilon = \frac{mg}{I} \sin\alpha (x_1 s - \int v dt) = \frac{mg}{I} \sin\alpha (x_1 s - vt)$ (6)

Integrating (6) with, we obtain the angular velocity: $\omega = \int \varepsilon dt = \int \frac{mg}{I} \sin\alpha (x_1 s - \int v dt) dt$
 $= \frac{mg}{I} \sin\alpha \left(x_1 s - \frac{vt^2}{2} \right)$ (7)

Let coefficient A represent constants in equation (7), we have:

$$A = mg \frac{\sin\alpha}{I}$$
 (8)

$$\omega = A \left(x_1 s t - \frac{vt^2}{2} \right)$$
 (9)

Integrating (9) with, we obtain the angle of the rotation:

$$\theta = A \left(\frac{x_1 s t^2}{2} - \frac{vt^3}{6} \right)$$
 (10)

Substituting $v=S/T$ into (10)

$$\theta = AS \left(\frac{x_1 t^2}{2} - \frac{t^3}{6T} \right)$$
 (11)

When the 1st half of cycle period of walking ends and then the next pair of legs start to be in standing phase ($t=T/2$), the angle of rotation around the supporting diagonal line

$$\theta = AST^2 \left(\frac{x_1}{8} - \frac{1}{48} \right)$$
 (12)

From (12), we can see that variable x_1 affects the angle of rotation around the supporting diagonal line. It is easy to see that the angle of rotation θ may be 0 if x_1 is equal to 1/6. Next we will analyze how does variable x_1 influence the angle of rotation and walking stability largely.

From (12), it can be seen that:

1. The angel of rotation θ is proportional to ST^2 . From $S = vt$ we can see that θ is proportional to T^3 with constant walking speed. In order to maintain attitude stability and walking stability, the smaller the cycle period and stride length, the better.
2. The angel of rotation θ is also proportional to coefficient A. From (8), A is proportional to $msin\alpha$ and inverse proportional to I. So a minor m and a rather large I are advantaged for decreasing the angel of rotation θ and reinforcing stability.
3. The initial position x ($x_1 S$) of quadruped walking machine largely influences attitude stability and walking stability of the robot. If x is $S/6$ ($x_1 = 1/6$), the angle of rotation at the end of supporting will be 0 theoretically (see Fig. 4) and the body of the robot is upright without any inclination. It is much advantaged for changing of supporting legs and dynamic stable walking.

If X is another value, such as 0, $S/4$ and so on, the angle of rotation around the diagonal supporting diagonal line is not 0, it will aggravate the collision with the ground and is not advantaged for dynamic stable walking [viii][ix].

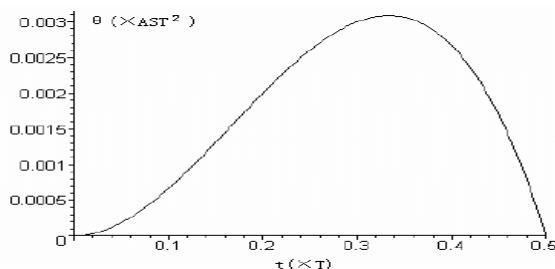


Fig (4): The angle of rotation of vehicle

IV. . SIMULATION RESULTS

To verify the analysis, MATLAB software is used to implement above model of quadruped walking machine 'Warp'. The geometrical data our paper [3] has been used for simulation studies. The cycle period T of the walk is 0.7 s. In order to study the effects of various initial stances on walking stability, different values of initial positions x are selected, such as 0, $S/6$ and $S/4$. By a series of simulations, we get data of roll angle (Rotation angle of vehicle along longitudinal axis) and pitch angle (The rotation angle alongside way of the vehicle) at $t = T/2$ in walking, the number of stable walking cycle periods and the status of variation of those angles in walking, Table 1 & Fig. 6.

From simulation results, at the end of the 1st half of cycle period of walking, variation of body attitude is the smallest if initial position x is $S/6$ (see Table 1. Here, it refers to the data of roll angle and pitch angle). Besides, the number of stable walking periods is the maximum in all initial positions, Table 1 & Fig. 6. The results illustrate that initial stance do influence walking stability of quadruped walking and $x = S/6$ is the optimal initial stance. Here, the optimal initial stance refers to the stance with minimal attitude varying at the end of the supporting ($t = T/2$) and maximal walking stability [x].

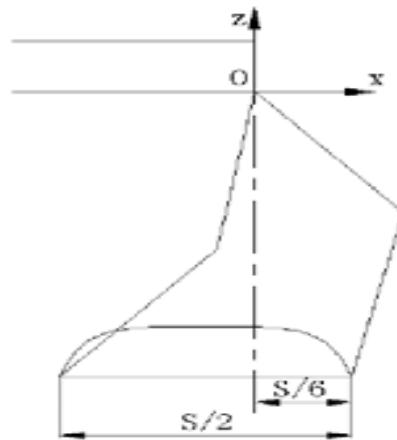
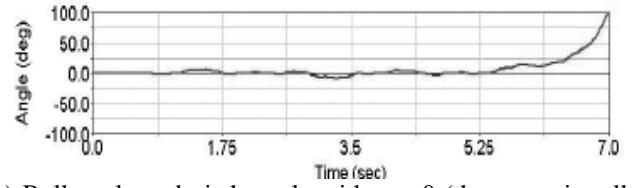
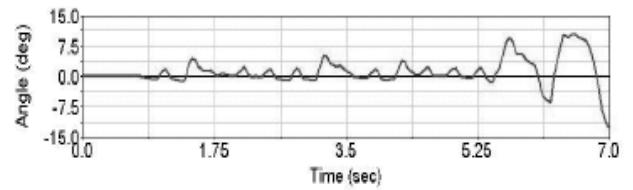


Fig (5): Initial stances diagonal legs on ground



(a) Roll angle and pitch angle with $x = 0$ (the upper is roll angle and the lower is pitch angle)

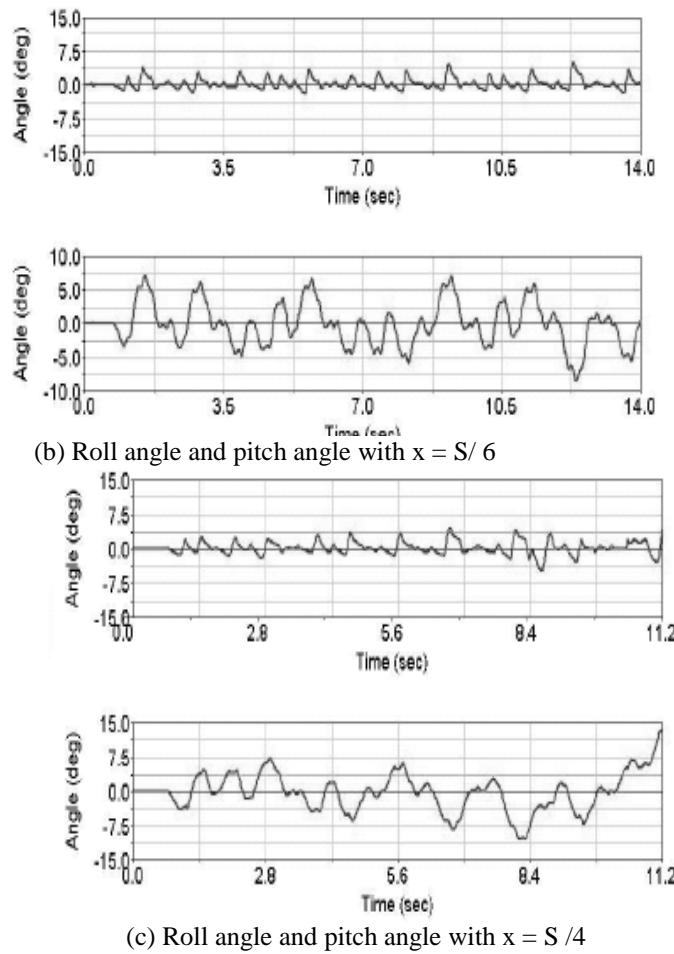


Fig. 6 Roll angle and pitch angle of the robot in walking

Table 1 Simulation results of different initial stances of quadruped trotting:

Initial position	Roll angle ($t = T/2$)	Pitch angle ($t = T/2$)	Number of stable periods
0	2.3*	1.7*	8
$S/6$	0.5*	3.1*	>19
$S/4$	0.7*	5.0*	15

V. CONCLUSION

In this paper we presented that initial stance of quadruped robot in trotting gait largely influences walking stability. Through equations and simulations we found that the angle of rotation around the supporting diagonal line of quadruped in trotting gait is minimal and walking stability is maximal if initial position x is $S/6$. The optimal initial stance also makes it easier to adjust and control body attitude of the robot. If proper control is utilized, quadruped walking machine will walk more quickly and stably.

It must also be pointed out that the value of optimal initial stance in this paper is true with constant walking speed. If there is any acceleration in walking, the optimal initial stance will change. But the method used in this paper could be used to get a new optimal initial stance.

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